Electron Collisions in Neon Plasma*

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The momentum transfer collision frequencies of electrons with neon ions and neutral atoms were measured in a decaying neon plasma over the temperature range from 200 to 600°K. The temperature-density dependence of the electron-ion collision frequency is determined to be

$$\nu_{\rm ei} = 3.6 \frac{n}{T_e^{3/2}} \ln \frac{2.0 \times 10^4 T_e^{3/2}}{n^{1/2}}.$$

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The energy dependence of the momentum transfer cross sections of electrons with neon atoms is best represented by $Q_{m_1}(u) = 2.55 \times 10^{-16} u^{1/2}$ and $Q_{m_2}(u) = 1.07 \times 10^{-17} + 2.17 \times 10^{-16} u^{1/2}$ cm² for one- and two-term approximately the sentence of the s proximations, where *u* is the electron energy in electron volts.

I. INTRODUCTION

HIS paper is concerned with the problems of the energy dependence of the momentum transfer cross section of electrons with neon atoms at low energies and the temperature dependence of the momentum transfer collision frequency of electrons with ions. these problems have been studied both theoretically 1-8 and experimentally⁹⁻¹³ by many authors. Generally, the few experimental values reported in the literature were not quite in harmony with one another. The present work was carried out in order to shed some light on these subjects. The method of using gaseous mixtures in studying various fundamental atomic processes is well known.^{14,15} In the present work, a decaying weakly ionized neon plasma at different fixed-temperature baths was examined by microwave interferometer and the neon neutral atoms and ions were treated as a

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¹ B. Kivel, Phys. Rev. **116**, 1484 (1959).

² B. L. Moiseiwitsch, Proc. Phys. Soc. (London) 77, 721 (1961); 81, 35 (1963).

³ T. F. O'Malley, Phys. Rev. 130, 1020 (1963).

⁴L. Landau, Phys. Z. Sowjetunion 10, 154 (1936).

⁵ S. Chandrasekhar, Principles of Stellar Dynamics (University of Chicago Press, Chicago, Illinois, 1942), p. 48.

⁶ V. L. Ginsburg, J. Phys. U.S.S.R. 8, 253 (1944).

⁷G. Burkhardt, G. Elwert and A. Unsöld, Z. Astrophysik 25, 310 (1948).

⁸ L. Spitzer, *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), p. 80.

9 A. V. Phelps, O. T. Fundingsland, and S. C. Brown, Phys. Rev. 84, 559 (1951). This is referred to as PFB in Fig. 2.

¹⁰ A. Gilardini and S. C. Brown, Phys. Rev. 105, 31 (1957).

¹¹ J. M. Anderson and L. Goldstein, Phys. Rev. 100, 1037 (1955).

¹² A. A. Dougal and L. Goldstein, Phys. Rev. 109, 615 (1958).

¹³ S. Takeda, J. Phys. Soc. Japan 16, 1267 (1961). ¹⁴ The pertinent references can be found in: H. S. W. Massey

and E. H. S. Burhop, Electronic and Ionic Inpact Phenomena (Clarendon Press, Oxford, England, London, 1952); S. C. Brown, Basic Data of Plasma Physics (Tech Press, Cambridge, Massachusetts and John Wiley & Sons, Inc., New York, 1959); H. J. Oskam, Philips Res. Rept. 13, 335 (1958), 13, 401 (1958); M. A. Biondi, Phys. Rev. 90, 730 (1953); M. A. Biondi and L. M. Chanin, *ibid.* 122, 843 (1961).

¹⁵ C. L. Chen, Phys. Rev. 131, 2550 (1963).

mixture in the decaying plasma with the density of one component (the ions) changing in the afterglow.

II. EXPERIMENTAL APPARATUS

The system used in the present investigation was essentially the same as that reported previously,¹⁵ and a detailed description of it will not be given here. In essence, the gas-handling system was of the standard bakeable ultrahigh-vacuum type,¹⁶ so that impurities introduced by the system were negligible. The gas used was mass spectrometrically controlled grade supplied by Linde Air Products Company. Furthermore, a cataphoresis pump¹⁷ was operated in series with the discharge tube continuously during the experiment. A transmission-type microwave ($\sim 2 \,\mu W$, 8.53 kMc/sec cw) interferometer^{11,15,18} was employed in measuring the electron densities n and the effective electron momentum transfer collision frequeicies ν_{eff} in the decaying neon plasma. The temperature of the discharge tube was monitored constantly by three copper-Constantan thermocouples. The temporal behavior of the electron temperature T_e in the very early part of the afterglow was also studied briefly by the X-band maser.¹⁵ It was found, in one case ($p_0 = 18.4$ Torr and $T_g = 300^{\circ}$ K), for example, that T_e dropped down to ~1000°K at 150 μ sec in the afterglow. After this time the decaying plasma was so transparent to the microwave that accurate measurements of T_e were impossible. Nevertheless, all our measurements (n and ν_{eff}) were made at times \sim 800 µsec or later in the afterglow. At such late times, it is believed that T_e had already relaxed to the gas temperature T_{g} . Since both electron-atom and electronion collisions are sensitive to electron temperature, at a given gas temperature the linear behavior of ν_{eff}/p^0 versus $n(0)/p_0$ (see next section) and its independence of gas pressure consititutes an indirect evidence to support this line of thought.

¹⁶ D. Alpert, J. Appl. Phys. 24, 860 (1953).

¹⁷ R. Riesz and G. H. Dieke, J. Appl. Phys. 25, 196 (1954).

¹⁸ L. Goldstein, M. A. Lampert, and R. H. Geiger, Elec. Commun. 29, 243 (1952).



FIG. 1. Measured $\nu_{\rm eff}(\rm Ne)/p_0(\rm Ne)$ versus $n(0)/p_0(\rm Ne)$ at two temperatures, i.e., 305 and 480°K. The straight-line behavior is predicted by Eq. (3) in the text under the condition of $\nu^2 \ll \omega^2$, where ω is the radian frequency of the applied field. The points are from: \blacksquare 3.66 mm Hg, \blacklozenge 3.78 mm Hg, \checkmark 6.52 mm Hg, \triangle 9.60 mm Hg, \bigcirc 2.28 mm Hg, + 4.49 mm Hg, * 8.92 mm Hg, \triangle 8.15 mm Hg. The intersection of $\nu_{\rm eff}(\rm Ne)/p_0(\rm Ne)$ at $n(0)/p_0=0$ gives $\nu_{\rm em}/p_0$ at that temperature and the slope is 0.675C(T_e).

III. RESULTS AND DISCUSSION

The effective electron collision frequency for momentum transfer $\nu_{\rm eff}$ is given by¹⁹

$$\nu_{\rm eff} = \nu_{\rm em} + \nu_{\rm ei} \tag{1}$$

for a weakly ionized gas under the influence of a lowlevel dc or rf electric field. Here $\nu_{\rm em}$ and $\nu_{\rm ei}$ are the momentum transfer collision frequencies of electrons with atoms and ions, respectively. Theoretical considerations of $\nu_{\rm ei}$,⁴⁻⁸ in general, lead to the functional form²⁰

$$\nu_{\rm ei} = C_1 \frac{n}{T_e^{3/2}} \ln \frac{C_2 T_e^{3/2}}{n^{1/2}}, \qquad (2)$$

where C_1 and C_2 are constants. The present investigation attempts (1) to establish the dependence of $\nu_{\rm ei}$ on T_e , and (2) to determine the constants C_1 and C_2 , experimentally. For a limited range of n, Eq. (2) may be written as

$$\nu_{\rm ei} = C(T_e)n,$$

where $C(T_e)$ is a function of T_e and only depends weakly on *n*. Since $\nu_{ei} \propto n$, a knowledge of spatial distribution of n is important in deducing $\nu_{\rm ei}$ from the experimentally measured $\nu_{\rm eff}$. The requirements that $T_e = T_g$ and that the spatial electron density distribution be known confine the study to times late in the afterglow where the electrons are well thermalized with the gas and the electron density decay is predominantly controlled by ambipolar diffusion of the fundamental mode.²¹ Under such conditions, the radial electron density distribution in the cylindrical discharge tube assumes the form of a zero-order Bessel function. A simple analysis and numerical integration pertaining to the present case²² give

$$\frac{\nu_{\rm eff}}{p_0} = \frac{\nu_{\rm em}}{p_0} + 0.675C(T_e) \frac{n(0)}{p_0}, \qquad (3)$$

where n(0) is the electron density at the axis of the tube and p_0 is the neon gas pressure referred to 0°C. Since $\nu_{\rm em}/p_0$ should be constant at a given temperature, Eq. (3) predicts a linear behavior of ν_{eff}/p_0 versus $n(0)/p_0$. Typical results at gas temperatures of 305 and 480°K are shown in Fig. 1. The intersection of $\nu_{\rm eff}/p_0$ at $n(0)/p_0 = 0$ gives ν_{em}/p_0 and the slope is 0.675 $C(T_e)$. One immediately notices from Fig. 1 that the electronion collisions are a significant percentage of ν_{eff} and the usual procedure¹⁵ of securing $\nu_{\rm em}/p_0$ from $\nu_{\rm eff}/p_0$ at late afterglow time is therefore not reliable in the present case and may lead to wrong interpretations.²³ $\nu_{\rm em}/p_0$ and 0.675 $C(T_e)$ so determined as a function of electron temperature are shown in Figs. 2 and 3, respectively. The velocity dependence of the momentum transfer cross section $Q_m(v)$ of electrons with neon atoms is determined from the best fit to the experimental data



FIG. 2. $\nu_{\rm em}({\rm Ne})/p_0({\rm Ne})$ versus $T_{\rm e}$. The dash and solid curves are the best fits to the experimental points according to Eq. (4) and the assumptions of $Q_{m_1}(v) = Bv$ and $Q_{m_2}(v) = A + Bv$, respectively.

²¹ M. A. Biondi and S. C. Brown, Phys. Rev. **75**, 1700 (1949); W. P. Allis and D. J. Rose, *ibid.* **93**, 84 (1954).

¹⁹ V. L. Ginsburgh and A. V. Gurevich, Usp. Fiz. Nauk **70**, 201 (1960) [English transl.: Soviet Phys.—Usp. **3**, 115 (1960)]; V. N. Kolesnikov and V. V. Obukhov-Denisov, Zh. Eksperim. i Teor. Fiz. **42**, 1901 (1962) [English transl.: Soviet Phys.—JETP **15**, 692 (1962)]; I. P. Shkarofsky, Can. J. Phys. **39**, 1619 (1961).

 $^{^{20}}$ In our case, the ions are singly charged and the ion density $n_i \cdot n.$

²² The constant 0.675 in Eq. (3) is derived from the interaction of electromagnetic wave (of TE_{10} mode) with the plasma in a cylindrical discharge tube housed coaxially in a square waveguide for the present experiment.

²³ C. L. Chen, Progress Report, Coordinated Science Laboratory, University of Illinois, Urbana, Illinois, April 1963 (unpublished).

bv^{15,19}

$$\frac{\nu_{\rm em}}{p_0} = 1.18 \times 10^{16} \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{m}{kT_e}\right)^{5/2} \\ \times \int_0^\infty Q_m(v) v^5 \exp\left[-\frac{mv^2}{2kT_e}\right] dv, \quad (4)$$

where m is the electron mass and v the electron velocity. k is Boltzmann's constant. It is found that both $Q_{m_1}(v) = 4.3 \times 10^{-24} v$ and $Q_{m_2}(v) = 1.07 \times 10^{-17} + 3.66$ $\times 10^{-24}$ v cm² give fairly good fits within the accuracy of the present experiments. The subscripts 1 and 2 represent one- and two-term approximations, respectively. In terms of electron energy u, in electron volts, they are

$$Q_{m_1}(u) = 2.54 \times 10^{-16} u^{1/2} \text{ cm}^2$$
 (5a)

and

$$Q_{m_2}(u) = 1.07 \times 10^{-17} + 2.17 \times 10^{-16} u^{1/2} \text{ cm}^2.$$
 (5b)

Equations (5a) and (5b) are plotted on Fig. 4. The experimental results of Gilardini and Brown¹⁰ at higher energies are also presented in Fig. 4. O'Malley³ recently has analyzed Ramsauer-Kollath's data of electron beam scattering experiments in noble gasses by "atomic effective range formulas."24 The parameters in his theory are so chosen to fit the Ramsauer-Kollath experimental cross sections and the calculations are extrapolated to zero energy. His neon result is reproduced in Fig. 4 and it is seen that all curves exhibit the same shape. The agreements of these results are considered to be excellent especially those of O'Malley's and the present one. Recently, the swarm experimental data of neon by



FIG. 3. $0.675C(T_e)$, electron-ion collision frequency per electron versus electron temperature. The dash line is the ex-expected $T_e^{-3/2}$ variation should the logarithm term be constant. The solid curve is the best fit to the experimental points according to Eq. (2).

²⁴ T. F. O'Malley, L. Spruch, and L. Rosenberg, J. Math. Phys. **2**, 491 (1961); Phys. Rev. **125**, 1300 (1962).



FIG. 4. Momentum transfer cross section of electrons with neon atoms. The result of the present experiment is compared with those found by Gilardini and Brown at higher energies and the theoretical calculations by O'Malley. The equivalent electron-volt values of kT_e at 200 and 600°K are also shown.

Pack and Phelps²⁵ have been analyzed by Frost and Phelps.²⁶ Their cross sections indicate a reasonable agreement with ours above ~ 0.03 eV to within a few percent.

Since the present values of the energy dependence of the momentum transfer collision cross section are deduced from $\nu_{\rm em}/p_0$ as a function of electron temperature, measurements of $\nu_{\rm em}/p_0$ at temperatures below 200°K are desirable, especially the question of whether neon is a Ramsauer gas or not is raised.27 No determinations of $\nu_{\rm em}/p_0$ below 200°K were made at the present time.²⁸ Nevertheless, some measurements of $\nu_{\rm eff}/p_0$ at a gas temperature of 77°K have been carried out by Dougal and Goldstein¹² (DG) in a study of a decaying neon plasma and by Marshall, Kawcyn, and Goldstein²⁹ (MKG) in a study of the widths of electron cyclotron resonance curves. Their results are, in general, higher than the extrapolated value at 77°K given by the present analysis. MKG pointed out that the high value of $\nu_{\rm em}/p_0$ at 77°K could not be conveniently explained by the theoretically predicted³⁰ existence of Ne₂ (with \sim 0.002-eV binding energy). Since the velocity of sound in neon³¹ indicated no noticeably change of the specific heat ratio down to 30°K and the compressibility of gaseous neon showed no molecular neon condensation with decreasing temperature.32 If DG and MKG's results are correct, they may constitute evidence for a low energy "Ramsauer-like" effect in neon. The pres-

²⁵ J. L. Pack and A. V. Phelps, Phys. Rev. 121, 798 (1961).
²⁶ L. S. Frost and A. V. Phelps (private communication).
²⁷ D. G. Thomson and H. S. W. Massey, Third International Conference on the Physics of Electronic and Atomic Collisions, University College London, London, 22-26 July 1963 (unpublished).

²⁸ A completely different temperature bath design is required for the experiments below 200°K. For lack of time, we could not accomplish this objective.

T. Marshall (private communication).
 D. ter Haar, W. M. Nicol, and M. P. Barnett, Physica 22, 911

(1956). ⁸¹ W. H. Keesom and J. H. van Lammeren, Physica 1, 1161 (1934). ⁸² L. Holborn and J. Otto, Z. Physik 33, 1 (1925); 38, 359 (1926).



FIG. 5. Scattering length versus atomic number determined by various authors in noble gases. * B. Kivel, see Ref. 1. \times , \circ T. F. O'Malley, see Ref. 3. \triangle C. L. Chen, see Ref. 1. \times , \circ T. F. O'Malley, see Ref. 3. \triangle C. L. Chen, see Ref. 1. \times (A. G. Engelhardt and A. V. Phelps, Phys. Rev. 133, A375 (1964), and private communication. + B. L. Moiseiwitsch, Proc. Phys. Soc. (London) 77, 721 (1961).

ent investigation indicates that if a Ramsauer-type minimum should exist for electron-neon scatterings, it would most probably be at an electron energy less than 0.01 eV.

In support of this, it is of interest to note that the scattering lengths for the five noble gases (i.e., He, Ne, Ar, Kr, and Xe) exhibit a reasonable linear relationship with respect to the number of electrons in the atom (see Fig. 5). Empirically, it is found that

$$A \simeq (1.58 - 0.158z)a_0,$$
 (6)

where A is the scattering length, z the atomic number and a_0 the electron Bohr radius. The agreements between values of A as determined by various authors are considered to be good, since almost all are extrapolated values. The apparent linear relationship of Ato z suggests that the scattering length for neon is very small. Should it be negative,²⁷ the energy E_m at which the Ramsauer minimum occurs would most probably be <0.003 eV estimated from Fig. 5 and using O'Malley's formula³ [Eq. (3.4)]. In view of this, DG and MKG's original data should be regarded as tentative. The theoretical interpretation of Eq. (6) is not clear at the present time. Should this trend be continued in the noble gas sequency to Rn, the scattering length of Rn would be $\simeq -12a_0$ and the value of $Q_m(0)$ would be $\simeq 5.05 \times 10^{-14}$ cm² to an accuracy of $\sim 20\%$.

To study the electron-ion collision frequency experimentally in an isothermal plasma, the physical conditions have to be compatible with the theoretical requirement that the electron and ion number densities in a Debye sphere are much greater than unity. To fulfill this condition, the electron density must be lower if the temperature is reduced. This condition, together with the electron temperature relaxation and electron density spatial distribution considerations, require that

TABLE I. Comparison of experimental and theoretical values of C_1 and C_2 .

Theory		Experiment	
C_1	C_2	C_1	• C ₂
3.62	14.7 ×10 ^{3 в}	3.6	3.7×10 ^{3 f}
4.84	12.5 ×10 ^{3 b}	3.6	20×10^{3} g
3.59	3.32×10 ³ °		
2.25	$8.4 \times 10^{3} d$		
3.62	12.5 ×10 ³ °		

^a See Ref. 4. ^b See Ref. 5. ^o See Ref. 6. ^d See Ref. 7.

⁶ See Ref. 7. The energy equipartition time constant τ_{eq} derived in their theory is set equal to $[(2m/M)_{eq}]^{-1}$. Here *m* and *M* are the electron and ion mass, respectively. [See, for example, I. P. Shkarofsky, Can. J. Phys. **41**, 1753 (1963), Eq. (65)]. See Ref. 11

^g Present studies.

measurements be confined to the late afterglow times where the electron density loss occurs mainly through the fundamental amibipolar diffusion mode. The slope of $\nu_{\rm eff}/p_0$ versus $n(0)/p_0$ plot, which is 0.675 $C(T_e)$, is shown in Fig. 3 for different electron temperatures. The 200°K point is not shown here because of its uncertainty in being compatible with the physical conditions stated above. The electron density decays more rapidly in this case than in others during the first few hundred microseconds in the afterglow. This is presumably due to the more rapid electron-ion recombination³³ at lower temperatures. In order to get accurate measurable attenuations (≥ 0.1 dB), high gas pressures (>10 mm Hg) must be used and the electron density decay is not entirely diffusion controlled up to ~ 1 msec in the afterglow where relative accurate microwave attenuations could still be made. From the results shown in Fig. 3, it is clear that

$$\nu_{\rm ei} \propto T_e^{-3/2}$$

approximately. The deviation of the ln 0.675 $C(T_e)$ versus $\ln T_e$ graph from the minus-three-half power law is due to the logarithmic factor in Eq. (2). A best fit³⁴ to the experimental data according to Eq. (2) (the solid curve in Fig. 3) yields the following values of C_1 and C_2

$$C_1 = 3.6 \text{ cm}^3 - {}^{\circ}\text{K}^{3/2} - \sec^{-1}$$
 (7a)

and

$$C_2 = 2.0 \times 10^4 \text{ cm}^{-3/2} - {}^{\circ}\text{K}^{-3/2}.$$
 (7b)

A comparison of the present results with those of the theoretical and experimental studies by other authors is shown in Table I.

The variations in C_1 and C_2 by different authors results from the different approximations adopted in various stages of their treatment of the problem. From

³³ D. R. Bates, in Atomic and Molecular Processes, edited by D. R. Bates (Academic Press Inc., New York, 1963). ³⁴ Since the logarithmic factor is relatively insensitive to a

narrow range of variation of the electron density, the present data are so chosen that $6.2 \times 10^{10} < n < 9.6 \times 10^{10}$ cm⁻³ with a mean value of the spatial average electron density $\bar{n} \simeq 8 \times 10^{10}$ cm⁻³ used for the best fit calculation.

Table I it is seen that the present work favors the theories developed by Landau⁴ and by Spitzer.⁸ However, no attempt is made to compare the present results with the theory (of transport phenomena) by Cohen, Spitzer, and Routly³⁵ and by Spitzer and Harm³⁶ because of the different nature of the problem involved.³⁷ The values of C_1 and C_2 determined by Anderson and Goldstein¹¹ are extremely good considering that they

³⁵ R. S. Cohen, L. Spitzer, and P. McR. Routly, Phys. Rev. 80, 230 (1950).

³⁶ L. Spitzer and R. Härm, Phys. Rev. 89, 977 (1953).

³⁷ For a Lorentz gas, the dc resistivity η_L cannot simply be written as $\eta_L = m v_{\rm ei}/ne^2$. [See L. Spitzer, *Physics of Fully Ionized* Gases (Interscience Publishers, Inc., New York, 1956), p. 82.] It can be shown from the Boltzmann transport equation that, in the case of $Q_m(v) = bv^{-4}$

$\eta_L = m\nu/\pi ne^2$.

where $\nu = nb\langle v \rangle^{-3}$ may be identified as ν_{ei} and $\langle v \rangle = [8kT_e/\pi m]^{1/2}$ is the mean thermal velocity of the electrons. To equate η_L so obtained to Spitzer's (Refs. 35, 36) more rigorous derivation of η_L yields

$$v_{\rm ei} = 3.36 \frac{n}{T_{.3/2}} \ln \frac{1.25 \times 10^4 T_{e^{3/2}}}{n^{1/2}}$$

There is an apparent discrepancy of v_{ei} so derived to that derived from τ_{eq} . (Ref. 8) Because of the approximate nature in the treatment to deduce ν_{ei} from η_L , we think that it is improper to compare the present result with the dc resistivity theory.

only did the experiment at one temperature (i.e., 300°K). Yet the present experiment clearly demonstrates the temperature dependence of electron-ion collision frequency for momentum transfer in an isothermal plasma.

In conclusion, the method of microwave diagnosis a decaying weakly ionized neon plasma at different gas temperatures provides a unique means of studying the fundamental processes of electron-neon atom and electron-ion collisions at very low energies.

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Specific Heat of Mercury and Thallium between 0.35 and 4.2°K[†]

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The specific heat of mercury and thallium were measured between 0.35 and 4.2°K. In the normal state below 0.7° K the mercury results are given by: $C_n = 1.79T + 5.23T^3 \text{ mJ/mole deg}$. The coefficient α of the T^3 term corresponds to a value of the Debye parameter Θ_0 of 71.9°K. For temperatures higher than 0.7°K, the lattice specific heat deviates above the T³ law. A plot of $\Theta(T)$ is given. Below 0.6°K, the specific heat of thallium in the normal state is given by: $C_n = 1.47T + 4.03T^3 \text{ mJ/mole}$ deg. The corresponding value of Θ_0 is 78.5°K. Above 0.6° K, the lattice specific heat of thallium shows a deviation below the pure T^3 law, a result contrary to that found for most solids. This would imply a deviation in the dispersion curve above the linear portion. A similar effect was observed in the specific heat of graphite which was explained on the basis of bond-bending modes of vibration. It is suggested that similar modes may explain this behavior for thallium. In the superconducting state the specific heat of both materials can be represented by a sum of the normal lattice term and a superconducting electronic term C_{es} of the form $a\gamma T_c \exp(-bT_c/T)$. For mercury, values are obtained for a=15 and b=1.64 with $T_c=4.16$ °K; for thallium a=9 and b=1.52 with $T_c=2.38$ °K. In the case of thallium the critical field as a function of temperature $H_c(T)$ is determined, with $H_c(0) = 176.5$ G.

INTRODUCTION

EARLIER measurements in this laboratory,¹ as well as independent measurements by O'Neal *et al.*² indicate an anomaly in the low-temperature specific heat of indium. At very low temperatures the superconducting-state specific heat drops below the lattice specific heat as obtained from normal-state measurements. For a while there were indications that niobium showed a similar effect,³ but newer measurements⁴

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C. A. Bryant and P. H. Keesom, Phys. Rev. 123, 491 (1961).
 H. R. O'Neal, N. M. Senozan, and N. E. Phillips, *Proceedings* of the Eighth International Conference on Low Temperature Physics, edited by R. O. Davies (Butterworths Scientific Publications, Ltd., London, 1963).

⁸ A. T. Hirshfeld, H. A. Leupold, and H. A. Boorse, Phys. Rev. 127, 1501 (1962).

⁴ B. J. C. van der Hoeven and P. H. Keesom, Phys. Rev. 134, A1320 (1964).